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Emotional Responses to Symbolism

Laurie G. Buxton

Special difficulties often arise in reading mathematics because of the symbols and notation that are used. This is caused not only by the range of symbols and their density of meaning (interiority) but also by strong emotional responses raised by certain symbols or combinations. These feelings may reflect unpleasant memories of when the symbols were first encountered, but may even derive from an unease with the shape of some of them.

Much learning hinges upon the decoding of symbols, for it is mainly by means of written symbols that the knowledge the human race has accumulated is stored. Most of us learn satisfactorily to read our own language, though any of us can be confronted with passages of prose or poetry which we are able to translate from the written symbols to the spoken word, but cannot claim readily to comprehend. On the whole we remain comfortable when presented with a piece of our own written language whose symbols do not, with some reservations discussed below, occasion us disquiet. However, how are they regarded by someone who has not been able to learn to read? The range of unpleasant feelings is considerable. The mere sight of symbols of language will occasion fear, distaste, embarrassment, and shame. Anyone who has sought to teach an adult illiterate will confirm that this statement is not too strong. There is, in fact, a vicious circle whereby the emotional response to the symbols is such as to inhibit the individual's cognitive processes, which may in themselves be perfectly adequate to acquire the skill of reading. It is difficult to put oneself in the position of a non-reader (or even a pre-reader, though we have all passed through this stage). But once we introduce mathematical symbols, most of the population can be put precisely in this situation.

I shall describe three experiments on reactions to symbols, and hope then to offer explanations of two of them. The first was conducted with various groups of people most of whom knew some mathematics and had a generally positive attitude to the subject. The following statement was shown on a screen by an overhead projector:

$\phi(x)$ is continuous for $x = \xi$ if, given δ, $\ni$
$\varepsilon(\delta)$ s.t. $|\phi(x) - \phi(\xi)| < \delta$ if $0 \leqslant |x - \xi| \leqslant \varepsilon(\delta)$

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and the assembled company was asked to read it. "Reading" meant turning the written symbols into speech; it did not imply comprehension. No-one was able even to "bark the words," let alone penetrate the meaning, so they were in the position, relative to this passage, of a genuine non-reader.

The group were asked individually to record their *emotional* response on seeing the statement. Some said "mystified" or "double-dutch," but there were a number of replies in the area of "fear," "anxiety," or "apprehension." This type of reaction, in fact, prevents people even being willing to attempt understanding. We may assume that the symbolism of mathematics, despite its many advantages, can induce feelings inimical to learning the subject.

At this stage it is worth distinguishing between symbols and notation. By symbols I mean single characters, such as ξ, but by a notation I mean a grouping of such signs to convey a particular meaning. When we write (3,4) the symbols used are common ones, but the particular grouping of signs has a great deal of extra meaning (or to use a Skemp term, "interiority") not detectable by anyone who merely knows the separate signs. The effect of this is to render the apparently common place rather mysterious. This is one of the features of the language of mathematics that makes it inaccessible to so many.

Returning to our first example of mathematical writing, it may be that the use of Greek letters accounts for some of the negative reactions. The second series of experiments with groups of people illustrates this, though there are other factors at work as well. When offered the suggestion, "Plot the point (3,4)," most of the groups I was dealing with were happy enough, in that they understood the notation and the instruction was clear to them. Not all were sure which way one should measure the 3 and which the 4, but that was the only area of unease.

With the statement, "Consider the point (x,y)," there was a sense of uncertainty and insecurity in some, deriving partly from the formality of the language and partly from the familiar numerals being replaced by slightly mysterious letters. Yet the statement was still on the whole acceptable.

Finally the group was presented with "Let $P(\xi,\eta)$ be such that . . . ." Quite apart from the unfinished nature of the statement and the increased formality of style, the impact of $P(\xi,\eta)$ was such as to render extinct any hope that what followed might be understood. One person claimed that once such a statement was stated, "The shutters came down" as far as he was concerned. In part this derived from the notation of setting the letter P next to the known notation, and in part from the Greek letters, which not everyone could even *say*.

Enough has perhaps been said to establish that the written language of mathematics has not only a density of meaning that renders understanding slow in coming but that the mere presentation induces an unease that will not allow one to make a start on penetrating the meaning. Those feelings laid down as a result of earlier failures in dealing with mathematical symbolism inhibit an appropriate cognitive attempt even at reading them.

There is another separate response that is of interest. In this third experiment, and working again with various groups of people, another emotional response to symbols has manifested itself. The evidence given so far suggests that unfamiliarity with Greek letters may be an important influence in producing negative reactions. Without refuting this, experiments in attitudes to single letters indicate that some are more acceptable than others and that certain Greek letters (such as α and ρ) are found to be pleasantly formed and quite appealing—and that this is not true of all the letters in our own alphabet. In presenting this issue to a group I discussed the fact that some relatively unfamiliar letters, such as α, did not seem to create unease. I then asked that the question of familiarity or frequency be cast from their minds and that each person should decide individually which lower-case letter of the English alphabet they found most *strange*. With every group of people, q emerges as the easy winner. Even with groups of as many as thirty people not more than six letters were mentioned, with x, z, k, and j appearing, but in every case with far fewer votes than q. It is not easy to guess what this may mean. Perhaps it is simply the shape. Certainly among the Greek letters ξ is not as easy to accept as ρ. Why should we respond in this way, and what effect does it have on our being able to deal with mathematics? At this stage I have not even reached a hopeful speculation.

So we see that all purely cognitive approaches to the understanding of mathematical symbols and notation will be ineffective unless they recognise that an emotional dimension exists. *Acceptance* of a symbol or a notation is an emotional issue. It may come simply with usage and familiarity, but mere definition will never suffice. Even with signs to which we have become accustomed there may remain a flavour of distaste which makes us less competent in their use.

So far the case is stated. If we now accept that there is a problem we have two things to do. The first is to give a rationale for why we should feel as these experiments indicate that we do, and the second is to suggest possible ways of preventing "symbol-fear" from arising, or (more difficult) remedying it when it has occurred.

Skemp (1980) has indicated that emotions may signal danger and an attack upon oneself. May we interpret the situation of reading a string of symbols in the light of this belief? Is an attack being made? Certainly all ones early learning experience leads one to believe that demands are being made. When presented with symbols, including the language ones, the demands is that some response be made, such as reading it, understanding it, or working something out. The threat lies at the end of that process, because failure to satisfy the teacher's wishes can often have a negative emotional outcome! Even if in a present situation no demands are being made, one's belief is that they are, based on previous experience. It may be that the symbols of mathematics make more difficult and heavier demands than other symbols. They are perhaps more functional, operational, active than the letters in which our prose is written. This is illustrated by the successive lines in which an equation is solved. So routine can become the various transpositions which we make that the symbols seem to have a life of their own in arriving at an answer. Even so, an illiterate probably does get quite strong negative charges from the printed work, and as we have said, most people are illiterate when it comes to "reading" mathematics.

The answer is not to avoid mathematical symbols in a child's earlier experience. Rather one should capitalise on situations where the children feel a need for symbols. Several examples may illustrate this.

A group of children in one primary school were playing with some chime bars and at the end of one day had found a tune which happened to please them all. They wanted to play it the next day. No sophisticated derivation of musical notation arose, but the five chimes were labelled *A, B, C, D, & E* and they simply wrote down a string of letters. The beginnings of a notation, and perhaps the first step towards algebra? Certainly a code they all understood, and not discoverable from outside without further information.

A second case, again with primary children, arose in recording journeys. In going from, say, school to public library at every junction they used one of three symbols L, R, or A. "A" stood for "ahead," "L" for "turn left," and "R" for "turn right." They worked happily with them and managed to establish how a string of symbols was transformed on the reverse journey. (Interestingly, if we introduce "U" for "about turn," we have a group isomorphic to that formed by the powers of i).

A last example comes from my own work with a group of experienced primary teachers. We were engaged upon an investigation of the regions created when straight lines cut each other, all in distinct points. We had arrived at a four-line configuration and at first I had labelled

the spaces by a string of capital letters. This was readily accepted. Suddenly I wanted to convey the number of boundaries of a region, and whether it was open or closed, and without preparation labelled it as shown in Figure 1. The effect of one member was to say that she was totally lost. The *need* for a notation should have been raised and the group should have been asked for suggestions.

Regular sessions, then, at all stages in mathematical education, of experiencing the need for a symbol or a notation and the discussion of the notation suggested, would greatly ease the situation. In most people's experience each addition to our complicated system has simply been produced like a rabbit out of a hat.

There is a related but distinct reaction to symbols that we may explain differently and perhaps remedy in other ways. A page of mathematics can induce not so much a clear and remembered threat as a feeling of insecurity, sometimes at a level that can be described as panic. A model of this feeling developed by Skemp and myself is described by myself (Buxton 1981). Briefly, a failure to comprehend the symbols places the reader in one area where is lost, with no sense of goals or direction, and with no sense of how to act appropriately. Panic ensues.

In general a symbol represents a concept, whereas a notation involves a whole schema lying behind it. We saw this in the plotting of a point described in various ways earlier. If the schema is not known to

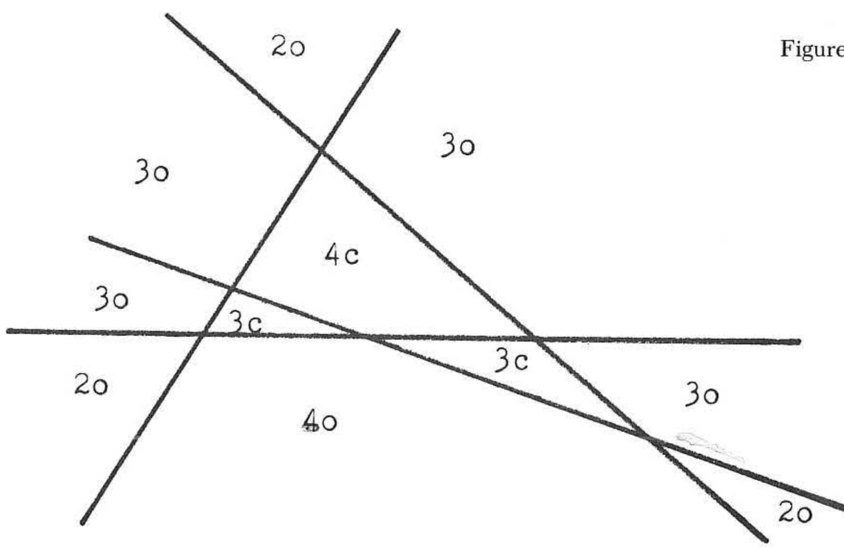

Figure 1.

the student, even if the separate concepts are, he will be unable to operate. Approaches to mathematics teaching that are largely content based will attempt to develop it logically and to develop all parts of the *subject* in an ordered fashion — and this is admirable. However, more important than the schema lying within the subject (in Popper's world 3) are those in the mind of the student (world 2) (Popper 1973). We need to check with the students whether they find that the information conveyed fits what they have in their minds. An interesting experiment is to ask a number of people whether "minus times a minus is a plus" fits comfortably into their minds. When the student believes, rightly or wrongly, that the idea does fit, then and only then should you move on. It is the "emotional acceptability" of what we are told or read that is the measure of whether we can advance.

Most teachers check out whether their students understand, and by this they are addressing the cognitive. It is necessary to ask whether they accept — and *that* is affective. Once the strings of symbols are attached comfortably to those patterns we already have in our minds, we are secure.

Finally we should mention one counter-indication to what we have said, and point again to one question discussed earlier but not resolved. In the discussion on (ξ,η) we did assume that the schema of two coordinate axes, and the plotting of points was known and comfortable. Why did the use of unfamiliar symbols induce discomfort? Perhaps it is felt that they must convey something more, something mysterious — else why were such letters used? But the reason is not clear.

As for why q is so "strange" — perhaps someone can help?

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Mathematical Language and Problem Solving

Gerald A. Goldin

Problem solving in mathematics may require different kinds of language: the verbal or mathematical language in which the problem itself is posed, the notational language of problem representations available to the solver, and planning language for heuristic reasoning and formulation of strategies. This paper explores some relationships among these languages, with examples of ways they can influence problem-solving processes.

I Introduction

Problem solving in mathematics refers to situations in which some items of information are given or available, and one or more goals are described. The problem solver is expected to attain the goal(s) through logical or mathematical procedures. Sometimes the term "problem solving" is restricted to the case in which the solver has no routine algorithm available for this purpose. Mathematics educators have become increasingly interested in studying problem solving and improving its teaching (Polya 1962 and 1965; Harvey & Romberg 1980; Krulik 1980; Lester 1980).

Kilpatrick (1978) proposed to organize the independent variables of problem-solving research into three main categories — subject variables, task variables, and situation variables — for the purpose of understanding how problem-solving outcomes depend on variables in each category. A collaborative study of task variables was conducted by a number of researchers (Goldin & McClintock 1979). In this work the characteristics of problem tasks were considered under the following headings: syntax variables, describing the grammar and syntax of the problem statement; content and context variables, describing the semantics of the problem statement; structure variables, describing mathematical aspects of a problem representation; and heuristic behavior variables, describing heuristic processes associated with or intrinsic to specific problems. Task variables were taken to be independent of the individual problem solver, and defined instead with respect to a population of solvers. They are subject in principle to control by the researcher or the teacher.

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